Substitution of equations (18) and (30) into equation (16) and equating the coefficients of equal powers of ε yield

$$\tau_0 = \int_0^{x_f} \left(\frac{\partial U_0}{\partial \delta}\bigg|_{\delta=1} \right)^{-1} x_f \,\mathrm{d}x_f \quad (31)$$

$$\tau_1 = \int_0^{x_f} \left(\frac{\partial U_1}{\partial \delta}\bigg|_{\delta=1} \right)\left(\frac{\partial U_0}{\partial \delta}\bigg|_{\delta=1} \right)^{-2} x_f \,\mathrm{d}x_f \quad (32)$$

$$\tau_2 = \int_0^{x_f} \left[\left(\frac{\partial U_0}{\partial \delta}\bigg|_{\delta=1} \right)\left(\frac{\partial U_2}{\partial \delta}\bigg|_{\delta=1} \right) - \left(\frac{\partial U_1}{\partial \delta}\bigg|_{\delta=1} \right)^2 \right]\left(\frac{\partial U_0}{\partial \delta}\bigg|_{\delta=1} \right)^{-3} x_f \,\mathrm{d}x_f \quad (33)$$

$$\tau_3 = \int_0^{x_f} \left[2\left(\frac{\partial U_0}{\partial \delta}\bigg|_{\delta=1} \right)\left(\frac{\partial U_1}{\partial \delta}\bigg|_{\delta=1} \right)\left(\frac{\partial U_2}{\partial \delta}\bigg|_{\delta=1} \right) - \left(\frac{\partial U_0}{\partial \delta}\bigg|_{\delta=1} \right)^2\left(\frac{\partial U_3}{\partial \delta}\bigg|_{\delta=1} \right) - \left(\frac{\partial U_1}{\partial \delta}\bigg|_{\delta=1} \right)^3 \right]\left(\frac{\partial U_0}{\partial \delta}\bigg| \right)^{-4} x_f \,\mathrm{d}x_f \quad (34)$$

where equation (17) has been used. Evaluation of τ_0, τ_1 and τ_2 by the use of equations (16)–(29) one obtains

$$\tau_0 = \tfrac{1}{2}[(1+x_f)^2-1] \quad (35)$$

$$\tau_1 = \frac{1}{6(1+x_f)}[(1+x_f)^3-3(1+x_f)+2] \quad (36)$$

$$\tau_2 = \frac{-1}{45(1+x_f)^4}[(1+x_f)^6-5(1+x_f)^3+9(1+x_f)-5] \quad (37)$$

$$\tau_3 = \frac{-1}{7560(1+x_f)^7}[64(1+x_f)^9+315(1+x_f)^7-2058(1+x_f)^6 +4725(1+x_f)^5-6804(1+x_f)^4+4725(1+x_f)^3 +1350(1+x_f)^2-3717(1+x_f)+1400]. \quad (38)$$

Higher order solutions of U_i and τ_i may be obtained by the same procedure. However, algebraic manipulation is complicated.

RESULTS AND DISCUSSION

The effect of ε on the interface position is illustrated in Fig. 1. The departure from the quasi-steady state solution, i.e. zero-order solution, increases as Stefan number, ε, increases as well as x_f increases.

Table 1 shows the values of τ_0, τ_1, τ_2 and τ_3 for the values of normalized interface position up to $x_f = 5$. The values of τ_0, τ_1 and τ_2 are consistent with the result of Pedroso and Domoto [4]. The values of τ_3 are quite different from the values of τ_3 of [4], which are also listed in Table 1. The difference between the perturbation method of this communication and Pedroso and Domoto [4] method is the use of Landau transformation in this communication. Landau transformation makes the nonlinearity due to moving interface explicit. Therefore, perturbation method can be used in a straightforward manner.

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ON THE ANALYSIS OF CELLULAR CONVECTION IN POROUS MEDIA

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NOMENCLATURE

a,	wave number;
a_0,	critical wave number;
A,	parameter defined in equation (17);
c,	solute concentration (salinity);
$\bar{c}$,	mean horizontal concentration;
d,	porous layer thickness;
d_p,	characteristic pore length;
g,	gravitational acceleration;
H,	solute advection spectrum;
$H_{pq}^{(n)}$,	coefficient in the series expanded for H;
K,	permeability;
N,	number of terms in the series expanded for ψ and γ;
Pe,	Peclet number (Ud_p/κ_s);
Re,	Reynolds number (Ud_p/ν);
s,	number of terms in the series expanded for S;
S,	Rayleigh number $(\alpha_s g\Delta cKd/\nu\varepsilon\kappa_s)$;
S_0,	critical Rayleigh number;
S_{0s},	parameter defined in equation (9);
Sc,	Schmidt number (ν/κ_s);
U,	module of velocity vector;
x,	horizontal coordinate;
z,	vertical coordinate.

Greek symbols

α_s,	coefficient relating salinity with density;
γ,	salinity perturbation;
$\Gamma_{pq}^{(n)}$,	coefficient in the series expanded for γ;

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δ, boundary-layer thickness;
δ_{pq}, Kronecker's delta;
Δc, salinity difference between lower and upper boundaries of the porous layer;
ε, porosity;
η, small parameter for power series expansion;
κ_s, molecular diffusivity;
ν, kinematic viscosity;
ψ, stream function;
$\Psi_{pq}^{(n)}$, coefficient in the series expanded for ψ.

INTRODUCTION

DESTABILIZING density gradients in a saturated porous medium layer may result from salinity (solute concentration) or thermal gradients. Such phenomena may occur in cases of an artificial recharge or deep well injection of water with salinity or temperature different from that of the groundwater. Temperature gradients may also be induced in groundwater by geothermal activity.

A brief review of analytical, numerical and experimental studies concerning free thermal convection in porous media was presented by Palm *et al.* [1] and Straus [2].

Palm *et al.* [1] applied a series expansion method for analyzing the thermal convection in porous media. Their analysis was based on a perturbation expansion method developed by Kuo [3]. However their approach requires tedious hand calculations. Recently Straus [2] used a semi-numerical Galerkin technique for the solution of this problem. This method was first used in the study of thermal convection by Veronis [4]. The problem can be solved numerically [5, 6]. However the numerical methods usually require large quantities of computer time. At high Rayleigh numbers special numerical grids should be used for the calculation of transport processes through the boundary layers developed on top and bottom of the convection cell.

The aim of this stuay is to apply Kuo's approach for simple calculations of the parameters characterizing free convection in porous media.

The analysis refers to saline convection (resulting from salinity gradients). However, it can be applied to thermal convection as well.

Consider a horizontal infinite layer of saturated porous material whose horizontal impermeable boundaries are at $z = 0$ and $z = d$ at which the salinities are constant. The equations of motion and diffusion can be nondimensionalized by applying the parameters d, Δc, κ_s/d, d^2/κ_s as characteristic length, salinity, velocity and time, respectively.

The motionless basic state is characterized by linear salinity and parabolic pressure profiles.

FINITE AMPLITUDE ANALYSIS

According to Schlüter *et al.* [7] two dimensional motion is the only stable mode for moderately supercritical Rayleigh numbers when thermal convection is conducted in a viscous fluid layer. Palm *et al.* [1] stated that nearly identical proof shows that this is true also for convection in a saturated porous layer. Straus [2] showed that the two dimensional mode is stable up to Rayleigh numbers ten times larger than the critical Rayleigh number when thermal convection is conducted in the porous layer.

The dimensionless Boussinesq equations of motion (Darcy) and diffusion governing the two dimensional perturbations are:

$$-S\frac{\partial\gamma}{\partial x}+\nabla^2\psi = 0 \qquad (1)$$

$$\nabla^2\gamma+\frac{\partial\psi}{\partial x} = H(\psi,\gamma) \qquad (2)$$

where the solute advection spectrum H is defined by

$$H(\psi,\gamma) = -\frac{\partial(\psi,\gamma)}{\partial(x,z)} = \frac{\partial\psi}{\partial z}\frac{\partial\gamma}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\gamma}{\partial z}. \qquad (3)$$

ψ and γ are the stream function and salinity perturbations. S is the solute Rayleigh number.

We intend to solve (1) and (2) with the following boundary conditions

$$\psi,\gamma = 0 \quad \text{at} \quad z = 0, 1 \qquad (4)$$

i.e. impermeable boundaries on which the salinity is constant.

According to the technique developed by Kuo [3], ψ and γ can be expanded in power series whose terms are again expanded in double Fourier series as follows:

$$\psi = \sum_{n=1}^{N}\sum_{p,q=1}^{\infty}\Psi_{pq}^{(n)}\sin pax\sin q\pi z\eta^n \qquad (5)$$

$$\gamma = \sum_{n=1}^{N}\sum_{\substack{p=0\\q=1}}^{\infty}\Gamma_{pq}^{(n)}\cos pax\sin q\pi z\eta^n \qquad (6)$$

where

$$\eta^2 = (S-S_0)/S. \qquad (7)$$

The Rayleigh number can be expanded in a finite power series as follows:

$$S = S_0+S_{0s}\sum_{j=1}^{s}\eta^{2j} \qquad (8)$$

where:

$$S_{0s} = S_0/(1-\eta^{2s}). \qquad (9)$$

If the analysis is conducted for $N = 1$, it collapses to linear stability analysis. Then a single term is sufficient for expressing ψ and γ. We may introduce $\Psi_{11}^{(1)} = A$, $\Gamma_{11}^{(1)} = A/2\pi$. Linear stability analysis yields the following critical values of the Rayleigh and wave numbers

$$S_0 = 4\pi^2 \qquad a_0 = \pi. \qquad (10)$$

There is a positive relationship between increases in Rayleigh numbers and wave numbers. However, taking the assumption that under supercritical conditions, the wave number remains constant, does not significantly affect the values of the Nusselt number obtained through the calculations [2]. Such an assumption is not required by the method used here but considerably simplifies the analysis.

Substituting (5), (6), and (8) in (1), (2), and (3) we obtain:

$$\pi^2(p^2+q^2)\Psi_{pq}^{(n)}-4\pi^3p\Gamma_{pq}^{(n)}-p\pi S_{0s}\sum_{i=1}^{\infty}\Gamma_{pq}^{(n-2i)} = 0 \qquad (11)$$

$$\pi^2(p^2+q^2)\Gamma_{pq}^{(n)}-p\pi\Psi_{pq}^{(n)}+H_{pq}^{(n)} = 0 \qquad (12)$$

where

$$\begin{aligned}H_{pq}^{(n)} = {}&(1-\tfrac{1}{2}\delta_{p0})\frac{\pi^2}{4}\sum_{i=1}^{n-1}\sum_{k,m=1}^{i}\Psi_{km}^{(i)}\\ &\times[(kq+mp)(\Gamma_{p-k,q+m}^{(n-i)}+\Gamma_{k-p,q+m}^{(n-i)}-\Gamma_{k+p,q-m}^{(n-i)}\\ &+\Gamma_{k+p,m-q}^{(n-i)})+(kq-mp)(-\Gamma_{p-k,q-m}^{(n-i)}+\Gamma_{p-k,m-q}^{(n-i)}\\ &-\Gamma_{k-p,q-m}^{(n-i)}+\Gamma_{k-p,m-q}^{(n-i)}+\Gamma_{k+p,q+m}^{(n-i)})].\end{aligned} \qquad (13)$$

According to (12) and (13)

$$\Gamma_{0q}^{(n)} = -H_{0q}^{(n)}/(\pi^2q^2). \qquad (14)$$

For $p \neq 0$ we obtain:

$$\Psi_{pq}^{(n)}\left[\frac{(p^2+q^2)^2-4p^2}{4p(p^2+q^2)}\right] = \frac{S_{0s}}{4\pi}\sum_{i=1}^{\infty}\Gamma_{pq}^{(n-2i)}-\frac{1}{\pi(p^2+q^2)}H_{pq}^{(n)}. \qquad (15)$$

When $p = q = 1$, (15) yields:

$$\Gamma_{11}^{(n)}+\sum_{i=1}^{\infty}\Gamma_{11}^{(n-2i)} = (2/S_{0s})H_{11}^{(n+2)}. \qquad (16)$$

According to (13) and (14), $\Gamma_{02}^{(2)} = A^2/(16\pi)$. According to (13) and (16)

$$A = (2/\pi)(S_{0s})^{0.5}. \qquad (17)$$

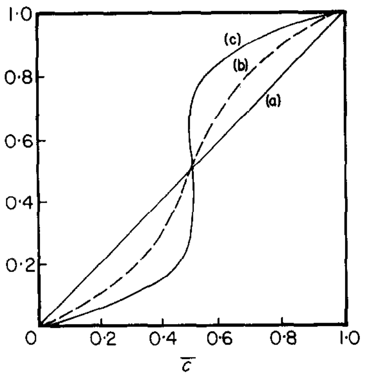

FIG. 1. Mean salinity profiles for various values of S/S_0. (a) Stable conditions, (b) $S/S_0 = 1{\cdot}5$, (c) $S/S_0 = 3{\cdot}0$.

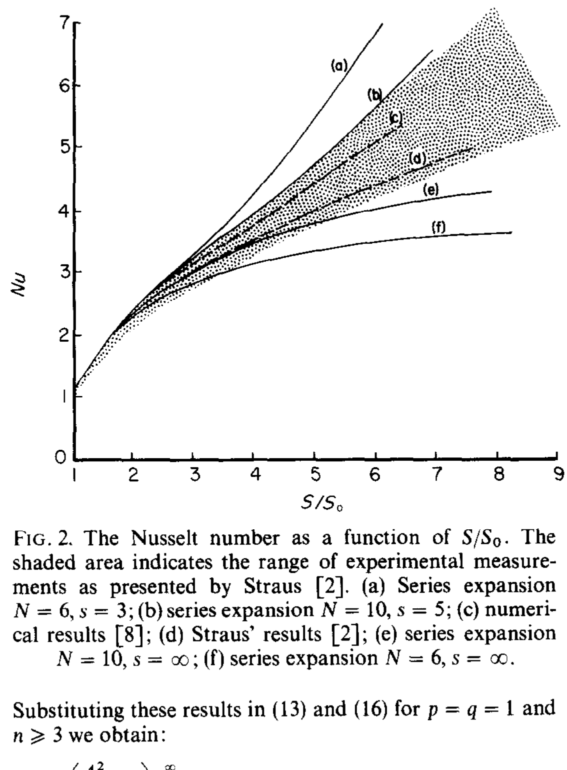

FIG. 2. The Nusselt number as a function of S/S_0. The shaded area indicates the range of experimental measurements as presented by Straus [2]. (a) Series expansion $N = 6, s = 3$; (b) series expansion $N = 10, s = 5$; (c) numerical results [8]; (d) Straus' results [2]; (e) series expansion $N = 10, s = \infty$; (f) series expansion $N = 6, s = \infty$.

Substituting these results in (13) and (16) for $p = q = 1$ and $n \geqslant 3$ we obtain:

$$\Gamma_{11}^{(n)} + \left(\frac{A^2}{16} - \frac{1}{2}\right) \sum_{i=1}^{\infty} \Gamma_{11}^{(n-2i)}$$
$$= \frac{1}{2\pi}\Psi_{13}^{(n)} + \tfrac{1}{2}\Gamma_{13}^{(n)} - \frac{1}{A^2}\sum_{i=3}^{n-1} \Psi_{km}^{(i)}\Big[(k+m)(\Gamma_{k+1,m-1}^{(n+2-i)} + \Gamma_{k-1,m+1}^{(n+2-i)}) + (k-m)(\Gamma_{k+1,m+1}^{(n+2-i)} + \Gamma_{k-1,m-1}^{(n+2-i)}) + \frac{kA}{2}(\Gamma_{k,2-m}^{(n+1-i)} - \Gamma_{k,2+m}^{(n+1-i)} - \Gamma_{k,m-2}^{(n+1-i)})\Big]. \quad (18)$$

For each value of n, the coefficients $\Gamma_{11}^{(n)}$ and $\Psi_{11}^{(n)}$ can be calculated directly through (18) after $\Psi_{13}^{(n)}$ and $\Gamma_{13}^{(n)}$ are known. All the coefficients $\Psi_{pq}^{(n)}$ and $\Gamma_{pq}^{(n)}$, except for $\Psi_{11}^{(n)}$ and $\Gamma_{11}^{(n)}$ can be calculated through (11), (13), and (15).

As the Rayleigh number increases, the series should include increasing numbers of terms. In such cases simple computer programming can be applied. Extremely short computer time is required even in the case of expansion including ten terms ($N = 10, s = 5$). The convergence of the method and the number of terms required for the series expansion can be measured by the convergence of the Nusselt number

$$Nu = \left.\frac{\partial \bar{c}}{\partial z}\right|_{z=0} = 1 + \pi \sum_{q,n=1}^{N} q\Gamma_{0q}^{(n)}\eta^n \quad (19)$$

where $\bar{c}$ is the mean horizontal salinity.

$$\bar{c} = z + \sum_{q,n=1}^{N} \Gamma_{0q}^{(n)} \sin q\pi z\eta^n. \quad (20)$$

In Fig. 1 we present profiles of $\bar{c}$ for various values of S/S_0. As expected this figure indicates the formation of boundary layers at the top and bottom boundaries as the Rayleigh number increases.

Figure 2 illustrates the variation of Nusselt number with Rayleigh number for various series expansions. Palm *et al.* [1], manually, calculated values of Nusselt number for $N = 6$ and $s = 3$. However, the convergence of the method is quite moderate and there are quite significant differences between the $N = 6$ and $N = 10$ expansions. Figure 2 indicates the range of experimental results as presented by Straus [2] and his analytical results. Straus' results lie in the lower part of the range of experimental values. The series expansion of $N = 10, s = 5$ yields results coinciding with the upper bound of the experimental values. The numerical results obtained with a very fine grid [8] are also shown in Fig. 2. These results are in fair agreement with the $N = 10$, $s = 5$ expansion as well as with Straus' results. However, it should be mentioned that better agreement could be attained by applying larger values of N.

DISCUSSION AND CONCLUSIONS

The analysis presented can be applied for the range of steady two dimensional free convection in a saturated porous layer (up to $S/S_0 \approx 10$). Series expansions of $N = 10$ can be used up to $S/S_0 \approx 7$. The advantages of this method are in its low requirement of computer time and simplicity.

We may follow boundary-layer approximations similar to those developed for the ordinary Bénard convection [10, 11] in order to analyze the limitations of the present study. Such an analysis yields the following expressions:

$$\left(\frac{d}{\delta}\right)^2 \approx S \qquad Nu \sim S^{1/2}$$
$$Pe \approx Sd_p/d \qquad Re \approx (S/Sc)(d_p/d). \quad (21)$$

These expressions indicate that at high Rayleigh numbers the Nusselt number is proportional to the square root of Rayleigh number (the $N = 10, s = 5$ expansion did not yield such a result). In such cases the boundary layer developed at the top and bottom of the convection cell has a thickness, δ (when applying numerical methods the grid mesh size at the convection cell boundaries should be smaller than δ) which can be of the same order of magnitude as the pore size. Then Darcy's law as well as the diffusion equation used in this study are not valid. At high Rayleigh numbers the Peclet number may attain large values. When the Peclet number is of the order of magnitude of unity, mechanical dispersion effects are of the order of magnitude of the molecular diffusion. Then the diffusion equation applied in this study is not valid. Moreover, at high Rayleigh numbers, the Reynolds number may be larger than unity. Therefore, Darcy's law is invalid. However, invalidity of the Darcy's law due to this condition is reasonable for thermal convection and less reasonable for saline convection.

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HEAT TRANSFER BY LAMINAR FILM CONDENSATION ON SPHERE SURFACES

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NOMENCLATURE

A, area [m^2];
c_p, heat capacity [J/kg K];
D, diameter of sphere [m];
g, gravitational acceleration [m/s^2];
h, heat-transfer coefficient [W/m^2 K];
$h'_{fg} = h_{fg} + 0{\cdot}68\, c_p(T_s - T_w)$, latent heat of vaporization corrected to account sensible heat of subcooling in the film after [5] [J/kg];
H, height of cylindrical part of vessel [m];
k, thermal conductivity [W/m K];
$\dot{m}$, mass rate of flow [kg/s];
Nu, Nusselt number;
Pr, Prandtl number;
$\dot{Q}$, heat flux [W];
R, radius of sphere surface [m];
$T_s - T_w$, difference between saturation temperature and wall temperature [K];
u, velocity in x direction [m/s];
x, coordinate measuring distance along circumference from the upper stagnation point of sphere [m];
y, coordinate measuring radial distance outward from sphere surface [m];
δ, thickness of the condensate film [m];
Θ, angular coordinate [rad];
μ, dynamic viscosity [kg/m s];
ρ, ρ_v, density of condensate and density of vapor [kg/m^3].

Subscripts
H, hemisphere;
0, initial conditions;
S, sphere;
$(\bar{\ })$, average value.

INTRODUCTION

IN CHEMICAL apparatus and devices of food industry, condensation processes very often occur on sphere surfaces. Mixer evaporators to condense vegetable or fruit pulps are an example of such devices.

The first results of calculations of heat transfer at film condensation on the sphere were given by Dhir and Lienhard [1]. They have used Nusselt's theory and have developed the general expression for the heat transfer coefficient on plane and axisymmetric bodies in nonuniform gravity. But this expression, obtained with the initial condition $\delta_0(x = 0) = 0$ for blunt bodies (such as the sphere, where $\delta_0 \neq 0$) formally is not valid. Recently Yang [2] has presented the results of numerical solution of momentum and energy equations, describing a thin layer of condensate in the form of laminar film running downward over the sphere. This communication presents the results of Nusselt's model analysis of heat transfer by laminar film condensation on sphere surfaces taking into account liquid wetting.

ANALYSIS

According to Nusselt's treatment [3] the downward flow of the condensate in the film, under the action of the gravity force, describes the balance equation between the gravity tangential component and the viscous forces $(\delta - y)(\rho - \rho_v)g \sin\Theta = \mu(du/dy)$, acting on the liquid element of volume $2\pi R \sin\Theta(\delta - y)R\,d\Theta$. The expression for the velocity distribution is

$$u(y) = (\rho - \rho_v)\mu^{-1} g \sin\Theta(\delta y - 0{\cdot}5y^2)$$

and the mass flow of condensate in the film through the section at the given angle Θ is:

$$\dot{m} = \int_0^\delta \rho u(y) 2\pi R \sin\Theta\, dy = \tfrac{2}{3}\pi\rho(\rho - \rho_v)\mu^{-1} g \sin^2\Theta R\delta^3. \quad (1)$$

As the flow of condensate proceeds from Θ to $\Theta + d\Theta$, the film thickness varies from δ to $\delta + d\delta$ as a result of both the influx of additional condensate and the change of the ring section area. This additional influx of condensate is

$$d\dot{m} = 2\pi\rho(\rho - \rho_v)\mu^{-1} g \sin\Theta R \times (\tfrac{2}{3}\delta^3 \cos\Theta\, d\Theta + \sin\Theta\delta^2\, d\delta). \quad (2)$$

The heat flux removed by the element of the sphere surface $dA = 2\pi R^2 \sin\Theta\, d\Theta$ must be equal to the incremental mass flow of condensate times the latent heat of condensation of the vapor:

$$dQ = k\frac{T_s - T_w}{\delta} dA = d\dot{m}h'_{fg} \quad (3)$$

hence after substituting equation (2) we obtain

$$(a\delta^4 \cos\Theta - b)\, d\Theta + \delta^3 \sin\Theta\, d\delta = 0 \quad (4)$$

where: $a = 2/3$, $b = \mu(T_s - T_w)Rk/\rho(\rho - \rho_v)gh'_{fg}$. This equation can be reduced to the complete differential equation. Therefore the solution of this equation in the form of $\delta = \delta(\Theta)$ can be obtained from the relation

$$\tfrac{2}{3}\delta^4 \int_{\Theta_0}^{\Theta} (\sin\Theta)^{5/3} \cos\Theta\, d\Theta - b\int_{\Theta_0}^{\Theta} (\sin\Theta)^{5/3}\, d\Theta + \tfrac{1}{4}(\sin\Theta_0)^{8/3}(\delta^4 - \delta_0^4) = 0$$

hence

$$\delta^4 = b\frac{\int_{\Theta_0}^{\Theta} (\sin\Theta)^{5/3}\, d\Theta + C}{\tfrac{1}{4}(\sin\Theta)^{8/3}}, \text{ where } C = \tfrac{1}{4}\frac{\delta_0^4}{b}(\sin\Theta_0)^{8/3}. \quad (5)$$

Let us consider two cases of the laminar film condensation on sphere and bottom hemisphere surfaces.